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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

May/June 2026

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n (4r-3)(12r+3) = an^3 + bn^2 + cn,$$

where a , b and c are integers to be determined.

[3]

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(b) Use the method of differences to find $\sum_{r=1}^n \frac{1}{(4r-3)(12r+3)}$ in terms of n .

[4]

(c) State the value of $\sum_{r=1}^{\infty} \frac{1}{(4r-3)(12r+3)}$.

[1]





2 Prove by mathematical induction that $3^{6n} + 92^n + 89$ is divisible by 91 for all positive integers n . [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



Turn over





- 3 The matrix $\mathbf{M}$ represents the sequence of two transformations in the x - y plane given by a stretch parallel to the x -axis, scale factor k ($k \neq 0, 1$), followed by a shear, x -axis fixed, with $(0, 1)$ mapped to $(2, 1)$.

(a) Show that $\mathbf{M} = \begin{pmatrix} k & 2 \\ 0 & 1 \end{pmatrix}$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) The transformation represented by $\mathbf{M}$ has a line of invariant points.

Find, in terms of k , the equation of this line. [3]

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The square S in the x - y plane is transformed by $\mathbf{M}$ onto the parallelogram P .

(c) Given that the area of S is half of the area of P , find the possible values of k . [2]

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4 The cubic equation $5x^3 - 2x + 1 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}$. [3]

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(b) Find the value of $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4}$. [3]

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(c) Find also the values of $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6}$ and $\frac{1}{\alpha^8} + \frac{1}{\beta^8} + \frac{1}{\gamma^8}$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



- 5** The lines l_1 and l_2 have equations $\mathbf{r} = -2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \mu(2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2 .

[5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



(b) The plane Π_1 contains l_1 and the point $P(1, -1, 1)$.

The plane Π_2 contains l_2 and passes through the origin.

Find the acute angle between Π_1 and Π_2 .

[6]

[illegible]



- 6 The curve C has equation $y = \frac{x^2 - x + 2}{4x + 4}$.

(a) Find the equations of the asymptotes of C .

[3]

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(b) Find the coordinates of any stationary points on C .

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(c) Sketch C .

[3]

(d) Sketch the curve with equation $y = \left| \frac{x^2 - x + 2}{4x + 4} \right|$.

[1]



[4]

[illegible]



7 The curve C has polar equation $r^2 = \frac{a}{1 + \tan 2\theta}$ for $0 \leq \theta < \frac{1}{4}\pi$, where a is a positive constant.

(a) Show that, at the point on C furthest from the initial line,

$$\cos \theta - \frac{\sin \theta}{\cos 2\theta (\sin 2\theta + \cos 2\theta)} = 0$$

and verify that this equation has a root between 0.5 and 0.6.

[6]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Sketch C .

[2]

(c) Use the substitution $v = \tan 2\theta$ to show that the area enclosed by C and the initial line is

$$\frac{1}{4}a \int_0^\infty \frac{1}{(1+v)(1+v^2)} dv$$

and find the exact value of this area.

[9]

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